Incomplete Descriptions and Qualified Definiteness

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Russell:

"Now *the*, when it is strictly used, involves uniqueness; we do, it is true, speak of "*the* son of So-and-so" even when So-and-so has several sons, but it would be more correct to say "*a* son of So-and-so". Thus for our purposes we take *the* as involving uniqueness." ([6]: 481)

Introduction

Definite descriptions

Form: ‘the F’

Strict use

‘the F’ is used strictly and a *complete definite description*, in case there is a unique F.

Example: ‘the pope’ in

(1.1) The pope is bald.

Loose use

‘the F’ is used loosely and an *incomplete definite description*, in case there is more than one F.

Example: ‘the bishop’ in

(1.2) The pope blesses the bishop.

Introduction

Aim

An intuitionistic *proof-theoretic semantics* for natural language constructions with incomplete definite descriptions.

Idea

Russellian analyses explain strict uses in 'the F is G' in terms of an existence, a uniqueness, and a predication clause:

(E) There is at least one F.

(U) There is at most one F.

(P) Every F is G.

Idea: replace the usual undefined notion of identity in the definition of uniqueness with the defined notion of *qualified identity* (W [7])

Framework

An intuitionistic *bipredicational natural deduction* system which combines components of [7] with the *rules for definiteness* proposed in Francez & W [1], [2].

Bipredicational first-order language $\mathcal{L}$

Permits two kinds of atomic predication:

- Predications (atomic formulae):

  $\varphi^n o_1 \ldots o_n$

- Negative predications (predication failures):

  $-\varphi^n o_1 \ldots o_n$

  ('the ascriptive combination of φ^n with $o_1, \ldots, o_n$ fails')

$\mathcal{L}$: Qualified identity

Let φ^n be an n-ary predicate constant.

$P^n_{\varphi^n}(o_1, o_2) =_{def}$

$$\forall z_1 \ldots \forall z_{n-1} \forall z_n \; ((\varphi^n o_1 z_2 \ldots z_n \leftrightarrow \varphi^n o_2 z_2 \ldots z_n) \\ \& \; (\varphi^n z_1 o_1 \ldots z_n \leftrightarrow \varphi^n z_1 o_2 \ldots z_n) \\ \& \; \ldots \; \& \; (\varphi^n z_1 \ldots z_{n-1} o_1 \leftrightarrow \varphi^n z_1 \ldots z_{n-1} o_2))$$

$N^n_{\varphi^n}(o_1, o_2) =_{def}$

$$\forall z_1 \ldots \forall z_{n-1} \forall z_n \; ((-\varphi^n o_1 z_2 \ldots z_n \leftrightarrow -\varphi^n o_2 z_2 \ldots z_n) \\ \& \; (-\varphi^n z_1 o_1 \ldots z_n \leftrightarrow -\varphi^n z_1 o_2 \ldots z_n) \\ \& \; \ldots \; \& \; (-\varphi^n z_1 \ldots z_{n-1} o_1 \leftrightarrow -\varphi^n z_1 \ldots z_{n-1} o_2))$$

Let $\varphi^{k_1}_1, \ldots, \varphi^{k_m}_m$ be all the predicate constants in $\mathcal{Q}$, where φ_i is k_i-ary and $\mathcal{Q} \subseteq \mathcal{P}$.

Positive qualified identity: $o_1 \overset{+}{=}_{\mathcal{Q}} o_2 =_{def} P^{k_1}_{\varphi_1}(o_1, o_2) \; \& \; \ldots \; \& \; P^{k_m}_{\varphi_m}(o_1, o_2)$

('o_1 is the same as o_2 in all $\mathcal{Q}$-respects')

Negative qualified identity: $o_1 \overset{-}{=}_{\mathcal{Q}} o_2 =_{def} N^{k_1}_{\varphi_1}(o_1, o_2) \; \& \; \ldots \; \& \; N^{k_m}_{\varphi_m}(o_1, o_2)$

('o_1 is the same as o_2 in no $\mathcal{Q}$-respect')

$\mathcal{L}\iota$: Qualified definiteness

We write $\varphi(x)$, suppressing the arity of φ, for atomic formulae $\varphi^n o_1 ... o_n$ containing (possibly multiple occurrences of) x. Let $\mathcal{Q} \subseteq \mathcal{P}$.

Positive qualified definiteness:

$$\psi(\iota_{\mathcal{Q}} x \varphi(x)) =_{def}$$
$$\exists x \varphi(x) \& \underbrace{\forall u \forall v ((\varphi(u) \& \varphi(v)) \supset u \overset{+}{=}_{\mathcal{Q}} v)}_{\text{Positive qualified uniqueness}} \& \forall w (\varphi(w) \supset \psi(w))$$

('the $\mathcal{Q}$-unique x which is φ is ψ'; simpler: 'the $\mathcal{Q}$-unique φ is ψ')

Negative qualified definiteness:

$$\psi(\iota_{\mathcal{Q}} x\text{-}\varphi(x)) =_{def}$$
$$\exists x\text{-}\varphi(x) \& \underbrace{\forall u \forall v ((\text{-}\varphi(u) \& \text{-}\varphi(v)) \supset u \overset{-}{=}_{\mathcal{Q}} v)}_{\text{Negative qualified uniqueness}} \& \forall w (\text{-}\varphi(w) \supset \psi(w))$$

('the $\mathcal{Q}$-unique x which fails to be φ is ψ'; simpler: 'the $\mathcal{Q}$-unique $-\varphi$ is ψ')

$\mathcal{L}\iota$: Maximal and restricted definiteness

Let $\mathcal{Q}' \subset \mathcal{P}$. Qualified definiteness has (i) the highest degree of definiteness in case $\mathcal{Q} = \mathcal{P}$ and (ii) a lower degree, in case $\mathcal{Q} = \mathcal{Q}'$. Given $\mathcal{Q}' \subset \mathcal{P}$, we can distinguish:

Maximal definiteness (=> complete definite descriptions):

- $\psi(\iota_{\mathcal{P}} x \varphi(x))$: 'the only x which is φ is ψ'
- $\psi(\iota_{\mathcal{P}} x - \varphi(x))$: 'the only x which fails to be φ is ψ'

Restricted definiteness (=> incomplete definite descriptions):

- $\psi(\iota_{\mathcal{Q}'} x \varphi(x))$: 'the x which is φ is ψ'
- $\psi(\iota_{\mathcal{Q}'} x - \varphi(x))$: 'the x which fails to be φ is ψ'

$\mathcal{L}\iota$: Negative predications with qualified descriptions

- $-\psi(\iota_{\mathcal{Q}} x \varphi(x))$: 'the $\mathcal{Q}$-unique x which is φ fails to be ψ'
- $-\psi(\iota_{\mathcal{Q}} x - \varphi(x))$: 'the $\mathcal{Q}$-unique x which fails to be φ fails to be ψ'

Bipredicational subatomic systems: Subatomic base

$\mathcal{S}_b$ is a pair $\langle \mathcal{I}, \mathcal{R}_b \rangle$, where $\mathcal{I}$ is a *subatomic base* and $\mathcal{R}_b$ a set of *I/E-rules for atomic sentences and negative predications*.

$\mathcal{I}$ is a 3-tuple $\langle \mathcal{C}, \mathcal{P}, v \rangle$, where v is such that:

- For any $\alpha \in \mathcal{C}$, $v : \mathcal{C} \to \wp(Atm)$, where $v(\alpha) \subseteq Atm(\alpha)$.
- For any $\varphi^n \in \mathcal{P}$, $v : \mathcal{P} \to \wp(Atm)$, where $v(\varphi^n) \subseteq Atm(\varphi^n)$.

Let $\tau\Gamma =_{def} v(\tau)$ for any $\tau \in \mathcal{C} \cup \mathcal{P}$. Call $\tau\Gamma$ the set of *term assumptions* for τ.

Bipredicational subatomic systems: Rules in $\mathcal{R}_b$

$$\dfrac{\begin{matrix}\mathcal{D}_0 \\ \varphi_0^n\Gamma\end{matrix} \qquad \begin{matrix}\mathcal{D}_1 \\ \alpha_1\Gamma\end{matrix} \quad \dots \quad \begin{matrix}\mathcal{D}_n \\ \alpha_n\Gamma\end{matrix}}{\varphi_0^n\alpha_1\dots\alpha_n}\,(as\mathrm{I}) \qquad \dfrac{\begin{matrix}\mathcal{D}_1 \\ \varphi_0^n\alpha_1\dots\alpha_n\end{matrix}}{\tau_i\Gamma}\,(as\mathrm{E}_i)$$

$$\dfrac{\begin{matrix}\mathcal{D}_0 \\ \varphi_0^n\Gamma\end{matrix} \qquad \begin{matrix}\mathcal{D}_1 \\ \alpha_1\Gamma\end{matrix} \quad \dots \quad \begin{matrix}\mathcal{D}_n \\ \alpha_n\Gamma\end{matrix}}{-\varphi_0^n\alpha_1\dots\alpha_n}\,(-as\mathrm{I}) \qquad \dfrac{\begin{matrix}\mathcal{D}_1 \\ -\varphi_0^n\alpha_1\dots\alpha_n\end{matrix}}{\tau_i\Gamma}\,(-as\mathrm{E}_i)$$

Side conditions:

1. $as\mathrm{I}$: $\varphi_0^n\alpha_1\dots\alpha_n \in \varphi_0^n\Gamma \cap \alpha_1\Gamma \cap \dots \cap \alpha_n\Gamma$.
2. $-as\mathrm{I}$: $\varphi_0^n\alpha_1\dots\alpha_n \notin \varphi_0^n\Gamma \cap \alpha_1\Gamma \cap \dots \cap \alpha_n\Gamma$.
3. $as\mathrm{E}_i$ and $-as\mathrm{E}_i$: $i \in \{0, \dots, n\}$ and $\tau_i \in \{\varphi_0^n, \alpha_1, \dots, \alpha_n\}$.

Terminology: We say that $-\varphi_0^n\alpha_1\dots\alpha_n$ is *negatively contained* in $\varphi_0^n\Gamma \cap \alpha_1\Gamma \cap \dots \cap \alpha_n\Gamma$, in case the side condition on $-as\mathrm{I}$ is satisfied.

Bipredicational subatomic identity systems: $\overset{+}{=}_{\mathcal{Q}}$-Rules

$\mathcal{S}_b^{=}$-systems extend $\mathcal{S}_b$-systems with I/E-rules for $\overset{+}{=}_{\mathcal{Q}}$ and $\overset{-}{=}_{\mathcal{Q}}$, where $\mathcal{Q} \subseteq \mathcal{P}$.

$$\dfrac{\begin{matrix}[\varphi_1(\alpha_1)]^{(1_1)} & [\varphi_1(\alpha_2)]^{(1_2)} & & [\varphi_k(\alpha_1)]^{(k_1)} & [\varphi_k(\alpha_2)]^{(k_2)} \\ \mathcal{D}_{1_1} & \mathcal{D}_{1_2} & & \mathcal{D}_{k_1} & \mathcal{D}_{k_2} \\ \varphi_1(\alpha_2) & \varphi_1(\alpha_1) & \ldots & \varphi_k(\alpha_2) & \varphi_k(\alpha_1)\end{matrix}}{\alpha_1 \overset{+}{=}_{\mathcal{Q}} \alpha_2} \; (\overset{+}{=}_{\mathcal{Q}}\mathrm{I}),\ 1_1, \ldots, k_2$$

$$\dfrac{\begin{matrix}\mathcal{D}_1 & \mathcal{D}_{i_1} \\ \alpha_1 \overset{+}{=}_{\mathcal{Q}} \alpha_2 & \varphi_i(\alpha_1)\end{matrix}}{\varphi_i(\alpha_2)} \; (\overset{+}{=}_{\mathcal{Q}}\mathrm{E}_i1) \qquad \dfrac{\begin{matrix}\mathcal{D}_1 & \mathcal{D}_{i_2} \\ \alpha_1 \overset{+}{=}_{\mathcal{Q}} \alpha_2 & \varphi_i(\alpha_2)\end{matrix}}{\varphi_i(\alpha_1)} \; (\overset{+}{=}_{\mathcal{Q}}\mathrm{E}_i2)$$

where $\varphi_i \in \mathcal{Q}$, $i \in \{1, \ldots, k\}$, and $\varphi_i(\alpha_1)$ and $\varphi_i(\alpha_2)$ are mirror atomic sentences.

Proof systems

Bipredicational subatomic identity systems: $\bar{=}_{\mathcal{Q}}$-Rules

$$\dfrac{\begin{matrix}[-\varphi_1(\alpha_1)]^{(1_1)} \\ \mathcal{D}_{1_1} \\ -\varphi_1(\alpha_2)\end{matrix} \quad \begin{matrix}[-\varphi_1(\alpha_2)]^{(1_2)} \\ \mathcal{D}_{1_2} \\ -\varphi_1(\alpha_1)\end{matrix} \quad \dots \quad \begin{matrix}[-\varphi_k(\alpha_1)]^{(k_1)} \\ \mathcal{D}_{k_1} \\ -\varphi_k(\alpha_2)\end{matrix} \quad \begin{matrix}[-\varphi_k(\alpha_2)]^{(k_2)} \\ \mathcal{D}_{k_2} \\ -\varphi_k(\alpha_1)\end{matrix}}{\alpha_1 \bar{=}_{\mathcal{Q}} \alpha_2}\ (\bar{=}_{\mathcal{Q}}\mathsf{I}),\ 1_1, \dots, k_2$$

$$\dfrac{\begin{matrix}\mathcal{D}_1 \\ \alpha_1 \bar{=}_{\mathcal{Q}} \alpha_2\end{matrix} \quad \begin{matrix}\mathcal{D}_{i_1} \\ -\varphi_i(\alpha_1)\end{matrix}}{-\varphi_i(\alpha_2)}\ (\bar{=}_{\mathcal{Q}}\mathsf{E}_i 1) \qquad \dfrac{\begin{matrix}\mathcal{D}_1 \\ \alpha_1 \bar{=}_{\mathcal{Q}} \alpha_2\end{matrix} \quad \begin{matrix}\mathcal{D}_{i_2} \\ -\varphi_i(\alpha_2)\end{matrix}}{-\varphi_i(\alpha_1)}\ (\bar{=}_{\mathcal{Q}}\mathsf{E}_i 2)$$

where $\varphi_i \in \mathcal{Q}$, $i \in \{1, \dots, k\}$, and $\varphi_i(\alpha_1)$ and $\varphi_i(\alpha_2)$ are mirror atomic sentences.

Bipredicational natural deduction systems

Rules of $\mathbf{I0}(\mathcal{S}_b^=)$-systems. Those of the above systems plus:

$$\dfrac{\begin{matrix}\mathcal{D}_1 & & \mathcal{D}_2\\ A & & B\end{matrix}}{A\&B}\,(\&\mathrm{I}) \qquad \dfrac{\begin{matrix}\mathcal{D}_1\\ A\&B\end{matrix}}{A}\,(\&\mathrm{E1}) \qquad \dfrac{\begin{matrix}\mathcal{D}_1\\ A\&B\end{matrix}}{B}\,(\&\mathrm{E2}) \qquad \dfrac{\begin{matrix}\mathcal{D}_1\\ A\end{matrix}}{A\vee B}\,(\vee\mathrm{I1}) \qquad \dfrac{\begin{matrix}\mathcal{D}_1\\ B\end{matrix}}{A\vee B}\,(\vee\mathrm{I2})$$

$$\dfrac{\begin{matrix} & [A]^{(u)} & [B]^{(v)}\\ \mathcal{D}_1 & \mathcal{D}_2 & \mathcal{D}_3\\ A\vee B & C & C\end{matrix}}{C}\,(\vee\mathrm{E}),\ u,v \qquad \dfrac{\begin{matrix}[A]^{(u)}\\ \mathcal{D}_1\\ B\end{matrix}}{A\supset B}\,(\supset\mathrm{I}),\ u \qquad \dfrac{\begin{matrix}\mathcal{D}_1 & & \mathcal{D}_2\\ A\supset B & & A\end{matrix}}{B}\,(\supset\mathrm{E})$$

$$\dfrac{\begin{matrix}\mathcal{D}_1\\ A(x/o)\end{matrix}}{\forall xA}\,(\forall\mathrm{I}) \qquad \dfrac{\begin{matrix}\mathcal{D}_1\\ \forall xA\end{matrix}}{A(x/o)}\,(\forall\mathrm{E}) \qquad \dfrac{\begin{matrix}\mathcal{D}_1\\ A(x/o)\end{matrix}}{\exists xA}\,(\exists\mathrm{I}) \qquad \dfrac{\begin{matrix} & [A(x/o)]^{(u)}\\ \mathcal{D}_1 & \mathcal{D}_2\\ \exists xA & C\end{matrix}}{C}\,(\exists\mathrm{E}),\ u$$

$$\dfrac{\begin{matrix}\mathcal{D}_1\\ \bot\end{matrix}}{A}\,(\bot\mathrm{i})$$

Bipredicational systems with qualified definiteness: $\iota_{\mathcal{Q}}$-Rules

$\mathbf{I0}(\mathcal{S}_b^=)\iota$-systems extend $\mathbf{I0}(\mathcal{S}_b^=)$-systems with I/E-rules for positive/negative qualified definiteness. $\mathcal{Q} \subseteq \mathcal{P}$.

Rules for positive qualified definiteness:

$$\frac{\begin{array}{c}\mathcal{D}_1\\ \exists x\varphi(x)\end{array} \qquad \begin{array}{c}\mathcal{D}_2\\ \forall u \forall v((\varphi(u)\ \&\ \varphi(v)) \supset u \overset{\pm}{=}_{\mathcal{Q}} v)\end{array} \qquad \begin{array}{c}\mathcal{D}_3\\ \forall w(\varphi(w) \supset \psi(w))\end{array}}{\psi(\iota_{\mathcal{Q}} x\varphi(x))}\ (\iota_{\mathcal{Q}}\mathrm{I})$$

$$\frac{\begin{array}{c}\mathcal{D}_1\\ \psi(\iota_{\mathcal{Q}} x\varphi(x))\end{array}}{\exists x\varphi(x)}\ (\iota_{\mathcal{Q}}\mathrm{E1}) \qquad \frac{\begin{array}{c}\mathcal{D}_1\\ \psi(\iota_{\mathcal{Q}} x\varphi(x))\end{array}}{\forall u \forall v((\varphi(u)\ \&\ \varphi(v)) \supset u \overset{\pm}{=}_{\mathcal{Q}} v)}\ (\iota_{\mathcal{Q}}\mathrm{E2})$$

$$\frac{\begin{array}{c}\mathcal{D}_1\\ \psi(\iota_{\mathcal{Q}} x\varphi(x))\end{array}}{\forall w(\varphi(w) \supset \psi(w))}\ (\iota_{\mathcal{Q}}\mathrm{E3})$$

(Likewise with $-\psi$.)

Bipredicational systems with qualified definiteness: $\iota_{\mathcal{Q}}$–-Rules

Rules for negative qualified definiteness:

$$\dfrac{\begin{matrix}\mathcal{D}_1 \\ \exists x - \varphi(x)\end{matrix} \qquad \begin{matrix}\mathcal{D}_2 \\ \forall u \forall v((-\varphi(u) \,\&\, -\varphi(v)) \supset u \,\bar{=}_{\mathcal{Q}}\, v)\end{matrix} \qquad \begin{matrix}\mathcal{D}_3 \\ \forall w(-\varphi(w) \supset \psi(w))\end{matrix}}{\psi(\iota_{\mathcal{Q}} x - \varphi(x))}\ (\iota_{\mathcal{Q}}\text{–I})$$

$$\dfrac{\begin{matrix}\mathcal{D}_1 \\ \psi(\iota_{\mathcal{Q}} x - \varphi(x))\end{matrix}}{\exists x - \varphi(x)}\ (\iota_{\mathcal{Q}}\text{–E1}) \qquad \dfrac{\begin{matrix}\mathcal{D}_1 \\ \psi(\iota_{\mathcal{Q}} x - \varphi(x))\end{matrix}}{\forall u \forall v((-\varphi(u) \,\&\, -\varphi(v)) \supset u \,\bar{=}_{\mathcal{Q}}\, v)}\ (\iota_{\mathcal{Q}}\text{–E2})$$

$$\dfrac{\begin{matrix}\mathcal{D}_1 \\ \psi(\iota_{\mathcal{Q}} x - \varphi(x))\end{matrix}}{\forall w(-\varphi(w) \supset \psi(w))}\ (\iota_{\mathcal{Q}}\text{–E3})$$

(Likewise with $-\psi$.)

Proof systems

Theorem: Normalization

Any derivation $\mathcal{D}$ in an $\mathbf{IO}(\mathcal{S}_b^=)\iota$-system can be transformed into a normal $\mathbf{IO}(\mathcal{S}_b^=)\iota$-derivation.

Theorem: Subexpression property

If $\mathcal{D}$ is a normal derivation of a unit U from a set of units Γ in an $\mathbf{IO}(\mathcal{S}_b^=)\iota$-system, then each unit in $\mathcal{D}$ is a subexpression of an expression in $\Gamma \cup \{U\}$.

Theorem: Subformula property

If $\mathcal{D}$ is a normal $\mathbf{IO}(\mathcal{S}_b^=)\iota$-derivation of formula A from a set of formulae Γ, then each formula in $\mathcal{D}$ is a subformula of a formula in $\Gamma \cup \{A\}$.

Internal completeness

J.-Y. Girard ([3]: 139-40):

> If we consider cut-free proofs, then all possible proofs are already there, there is no way to produce new ones. In other terms, the calculus is complete—nothing is missing. Observe that this completeness does not refer to any sort of model, it is an internal property of syntax. Such a property cannot be an accident, it should be given its real place, the first:
>
> *The subformula property is the actual completeness.*

Subatomic proof-theoretic semantics

Let I be an $\mathbf{I0}(\mathcal{S}_b^=)\iota$-system.

- The meaning of a *non-logical constant* τ is given by the term assumptions $\tau\Gamma$ for τ which are determined by the subatomic base of the $\mathcal{S}_b^=$-system of I.
- The meaning of a *formula* A of $\mathcal{L}\iota$ is given by the set of canonical derivations (i.e., derivations which use an I-rule in the last inference step) of A in I.

Example: A derivation of a $\iota_{\mathcal{Q}}$-sentence

Let $\mathcal{Q} = \{\varphi_1, \dots, \varphi_k\}$, $\mathcal{Q} \subseteq \mathcal{P}$, and $\varphi_i, \varphi_j \in \mathcal{Q}$, where $i, j \in \{1, \dots, k\}$ and $i \neq j$.

A derivation for (E):

$$\mathcal{D}_1 = \dfrac{\dfrac{\varphi_i\Gamma \qquad \dots \qquad \alpha\Gamma}{\varphi_i(\alpha)}}{\exists x \varphi_i(x)} \tag{1}$$

A derivation for (QU):

$$\mathcal{D}_2 = \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\varphi_1(\alpha)]^{(1_1)}}{\varphi_1\Gamma} \quad \dots \quad \dfrac{\dfrac{[\varphi_i(\alpha)\&\varphi_i(\beta)]^{(1)}}{\varphi_i(\beta)}}{\beta\Gamma}}{\varphi_1(\beta)} \qquad \dfrac{\dfrac{[\varphi_1(\beta)]^{(1_2)}}{\varphi_1\Gamma} \quad \dots \quad \dfrac{\dfrac{[\varphi_i(\alpha)\&\varphi_i(\beta)]^{(1)}}{\varphi_i(\alpha)}}{\alpha\Gamma}}{\varphi_1(\alpha)} \qquad \{\mathcal{D}\}}{\alpha \overset{\pm}{=}_{\mathcal{Q}} \beta}\; 1_1, \dots, k_2}{(\varphi_i(\alpha)\&\varphi_i(\beta)) \supset \alpha \overset{\pm}{=}_{\mathcal{Q}} \beta}\; 1}{\forall v((\varphi_i(\alpha)\&\varphi_i(v)) \supset \alpha \overset{\pm}{=}_{\mathcal{Q}} v)}\; \text{iii}}{\forall u \forall v((\varphi_i(u)\&\varphi_i(v)) \supset u \overset{\pm}{=}_{\mathcal{Q}} v)}\; \text{iii} \tag{2}$$

$\{\mathcal{D}\}$ in (2) is short for the set of the subderivations $\mathcal{D}_{2_1}, \mathcal{D}_{2_2}, \dots, \mathcal{D}_{k_1}, \mathcal{D}_{k_2}$ in applications of I-rules for qualified identity.

Example: A derivation of a $\iota_{\mathcal{Q}}$-sentence [contd.]

A derivation for (P):

$$\mathcal{D}_3 = \cfrac{\cfrac{\cfrac{\varphi_j\Gamma \quad \ldots \quad \cfrac{[\varphi_i(\alpha)]^{(2)}}{\alpha\Gamma}}{\varphi_j(\alpha)}}{\varphi_i(\alpha) \supset \varphi_j(\alpha)}\ 2}{\forall w(\varphi_i(w) \supset \varphi_j(w))}\ \text{iii} \tag{3}$$

A derivation of a $\iota_{\mathcal{Q}}$- (or QD-)sentence:

$$\cfrac{\begin{matrix}\mathcal{D}_1 \\ \exists x\varphi_i(x)\end{matrix} \quad \begin{matrix}\mathcal{D}_2 \\ \forall u\forall v((\varphi_i(u)\&\varphi_i(v)) \supset u \overset{\pm}{=}_{\mathcal{Q}} v)\end{matrix} \quad \begin{matrix}\mathcal{D}_3 \\ \forall w(\varphi_i(w) \supset \varphi_j(w))\end{matrix}}{\varphi_j(\iota_{\mathcal{Q}}x\varphi_i(x))}\ (\iota_{\mathcal{Q}}\mathrm{I}) \tag{4}$$

Complete definite descriptions: Examples

(1.1) The pope is bald.
$Bald(\iota_{\mathcal{P}} x Pope(x))$

(1.3) The king of France is not real.
$-Real(\iota_{\mathcal{P}} x(King\text{-}of^2(x, France)))$

Complete definite descriptions: Meaning

Let $\{\varphi_i\} \subset \mathcal{Q}' \subset \mathcal{P}$ and $\varphi_i, \varphi_j \in \mathcal{Q}'$, where $i, j \in \{1, ..., k\}$ and $i \neq j$.

Case (i): Maximal qualified definiteness

$$\mathcal{Q} = \mathcal{P}$$

$$\frac{\begin{array}{ccc} \mathcal{D}_1 & \mathcal{D}_{2(i)} & \mathcal{D}_3 \\ \exists x \varphi_i(x) & \forall u \forall v((\varphi_i(u) \& \varphi_i(v)) \supset u \overset{+}{=}_{\mathcal{P}} v) & \forall w(\varphi_i(w) \supset \varphi_j(w)) \end{array}}{\varphi_j(\iota_{\mathcal{P}} x \varphi_i(x))} \ (\iota_{\mathcal{P}}\mathrm{I}) \qquad (5)$$

(Strict use of ‘the φ_i’)

Incomplete definite descriptions: Examples

(1.4) The bishop is bald.
$Bald(\iota_{\mathcal{Q}}xBishop(x))$

Incomplete definite descriptions: Meaning

Let $\{\varphi_i\} \subset \mathcal{Q}' \subset \mathcal{P}$ and $\varphi_i, \varphi_j \in \mathcal{Q}'$, where $i, j \in \{1, ..., k\}$ and $i \neq j$.

Case (ii): Intermediate qualified definiteness

$$\mathcal{Q} = \mathcal{Q}'$$

$$\frac{\overset{\mathcal{D}_1}{\exists x \varphi_i(x)} \quad \overset{\mathcal{D}_{2(ii)}}{\forall u \forall v((\varphi_i(u) \& \varphi_i(v)) \supset u \overset{+}{=}_{\mathcal{Q}'} v)} \quad \overset{\mathcal{D}_3}{\forall w(\varphi_i(w) \supset \varphi_j(w))}}{\varphi_j(\iota_{\mathcal{Q}'}x\varphi_i(x))} \,(\iota_{\mathcal{Q}'}\mathrm{I}) \tag{6}$$

(Loose use of 'the φ_i')

Generic definite descriptions: Examples

(1.5) The Englishman is brave.
$Brave(\iota_{\{Englishman\}}xEnglishman(x))$

(1.6) The non-smoker is healthy.
$Healthy(\iota_{\{Smoker\}^{-}}x(-Smoker(x))$

Generic definite descriptions: Meaning

Let $\{\varphi_i\} \subset \mathcal{Q}' \subset \mathcal{P}$ and $\varphi_i, \varphi_j \in \mathcal{Q}'$, where $i, j \in \{1, ..., k\}$ and $i \neq j$.

Case (iii): Minimal qualified definiteness

$$\mathcal{Q} = \{\varphi_i\}$$

$$\frac{\overset{\mathcal{D}_1}{\exists x\varphi_i(x)} \qquad \overset{\mathcal{D}_{2(iii)}}{\forall u \forall v((\varphi_i(u) \& \varphi_i(v)) \supset u \stackrel{\pm}{=}_{\{\varphi_i\}} v)} \qquad \overset{\mathcal{D}_3}{\forall w(\varphi_i(w) \supset \varphi_j(w))}}{\varphi_j(\iota_{\{\varphi_i\}}x\varphi_i(x))} \; (\iota_{\{\varphi_i\}}\mathrm{I}) \qquad (7)$$

(Generic use of 'the φ_i')

Generalization A

Parallel QD: Examples

(1.2) The pope blesses the bishop.

(1.7) The dog descends from the wolf. (Cf. [5]: (33).)

(1.8) The pope puts the zucchetto on the zucchetto. (Cf. [5]: (38).)

Parallel QD: Extension of $\mathcal{L}\iota$

Let $\mathcal{Q}_1, ..., \mathcal{Q}_n \subseteq \mathcal{P}$.

- *Parallel positive QD*:

$$\begin{aligned}
&\psi(\iota_{\mathcal{Q}_1} x_1 \varphi_1(x_1), ..., \iota_{\mathcal{Q}_n} x_n \varphi_n(x_n)) =_{def} \\
&\qquad (\exists x_1 \varphi_1(x_1) \ \& \ ... \ \& \ \exists x_n \varphi_n(x_n)) \ \& \\
&\qquad (\forall u_1 \forall v_1((\varphi_1(u_1) \& \varphi_1(v_1)) \supset u_1 \overset{+}{=}_{\mathcal{Q}_1} v_1) \ \& \ ... \ \& \\
&\qquad\quad \forall u_n \forall v_n((\varphi_n(u_n) \& \varphi_n(v_n)) \supset u_n \overset{+}{=}_{\mathcal{Q}_n} v_n)) \ \& \\
&\qquad (\forall w_1 ... \forall w_n((\varphi_1(w_1) \ \& \ ... \ \& \ \varphi_n(w_n)) \supset \psi(w_1, ..., w_n)))
\end{aligned}$$

- *Parallel negative QD*:

$$\begin{aligned}
&\psi(\iota_{\mathcal{Q}_1} x_1 - \varphi_1(x_1), ..., \iota_{\mathcal{Q}_n} x_n - \varphi_n(x_n)) =_{def} \\
&\qquad (\exists x_1 - \varphi_1(x_1) \ \& \ ... \ \& \ \exists x_n - \varphi_n(x_n)) \ \& \\
&\qquad (\forall u_1 \forall v_1((-\varphi_1(u_1) \& -\varphi_1(v_1)) \supset u_1 \overset{-}{=}_{\mathcal{Q}_1} v_1) \ \& \ ... \ \& \\
&\qquad\quad \forall u_n \forall v_n((-\varphi_n(u_n) \& -\varphi_n(v_n)) \supset u_n \overset{-}{=}_{\mathcal{Q}_n} v_n)) \ \& \\
&\qquad (\forall w_1 ... \forall w_n((-\varphi_1(w_1) \ \& \ ... \ \& \ -\varphi_n(w_n)) \supset \psi(w_1, ..., w_n)))
\end{aligned}$$

(Likewise with $-\psi$.)

Generalization A

Parallel QD: Abbreviations

Let $\mathcal{Q}_k \subseteq \mathcal{P}$ with $k \in \{1, ..., n\}$.

1. *Abbreviations for parallel positive QD*:
   (a) E_k: $\exists x_k \varphi_k(x_k)$
   (b) QU_k: $\forall u_k \forall v_k((\varphi_k(u_k) \& \varphi_k(v_k)) \supset u_k \overset{+}{=}_{\mathcal{Q}_k} v_k)$
   (c) P: $\forall w_1 ... \forall w_n((\varphi_1(w_1) \& ... \& \varphi_n(w_n)) \supset \psi(w_1, ..., w_n))$
2. *Abbreviations for parallel negative QD*:
   (a) $-E_k$: $\exists x_k - \varphi_k(x_k)$
   (b) $-QU_k$: $\forall u_k \forall v_k((-\varphi_k(u_k) \& - \varphi_k(v_k)) \supset u_k \overset{-}{=}_{\mathcal{Q}_k} v_k)$
   (c) $-P$: $\forall w_1 ... \forall w_n((-\varphi_1(w_1) \& ... \& - \varphi_n(w_n)) \supset \psi(w_1, ..., w_n))$

   (Likewise with $-\psi$.)

Generalization A

Parallel QD: Rules

1. *Rules for parallel positive QD*:

$$\dfrac{\begin{matrix} \mathcal{D}_{1_1} & & \mathcal{D}_{1_n} & \mathcal{D}_{2_1} & & \mathcal{D}_{2_n} & \mathcal{D}_3 \\ E_1 & \dots & E_n & QU_1 & \dots & QU_n & P \end{matrix}}{\psi(\iota_{\mathcal{Q}_1} x_1 \varphi_1(x_1), \dots, \iota_{\mathcal{Q}_n} x_n \varphi_n(x_n))}\ (\iota_{\mathcal{Q}}\mathrm{I}_i^{\mathrm{A}})$$

$$\dfrac{\begin{matrix} \mathcal{D}_1 \\ \psi(\iota_{\mathcal{Q}_1} x_1 \varphi_1(x_1), \dots, \iota_{\mathcal{Q}_n} x_n \varphi_n(x_n)) \end{matrix}}{E_k}\ (\iota_{\mathcal{Q}}\mathrm{E}_k^{\mathrm{A}}1)$$

$$\dfrac{\begin{matrix} \mathcal{D}_1 \\ \psi(\iota_{\mathcal{Q}_1} x_1 \varphi_1(x_1), \dots, \iota_{\mathcal{Q}_n} x_n \varphi_n(x_n)) \end{matrix}}{QU_k}\ (\iota_{\mathcal{Q}}\mathrm{E}_k^{\mathrm{A}}2)$$

$$\dfrac{\begin{matrix} \mathcal{D}_1 \\ \psi(\iota_{\mathcal{Q}_1} x_1 \varphi_1(x_1), \dots, \iota_{\mathcal{Q}_n} x_n \varphi_n(x_n)) \end{matrix}}{P}\ (\iota_{\mathcal{Q}}\mathrm{E}_i^{\mathrm{A}}3)$$

where $i \in \{1, \dots, n\}$ (arity of ψ), $k \in \{1, \dots, n\}$

(Likewise with $-\psi$.)

Parallel QD: Rules [contd.]

2. *Rules for parallel negative QD*:

$$\frac{\begin{array}{ccccc} \mathcal{D}_{1_1} & \mathcal{D}_{1_n} & \mathcal{D}_{2_1} & \mathcal{D}_{2_n} & \mathcal{D}_3 \\ -E_1 \;\ldots\; -E_n & & -QU_1 \;\ldots\; -QU_n & & -P \end{array}}{\psi(\iota_{\mathcal{Q}_1}x_1 - \varphi_1(x_1), \ldots, \iota_{\mathcal{Q}_n}x_n - \varphi_n(x_n))}\;(\iota_{\mathcal{Q}-}\mathsf{I}^{\mathrm{A}}_i)$$

$$\frac{\begin{array}{c}\mathcal{D}_1\\ \psi(\iota_{\mathcal{Q}_1}x_1 - \varphi_1(x_1), \ldots, \iota_{\mathcal{Q}_n}x_n - \varphi_n(x_n))\end{array}}{-E_k}\;(\iota_{\mathcal{Q}-}\mathsf{E}^{\mathrm{A}}_k1)$$

$$\frac{\begin{array}{c}\mathcal{D}_1\\ \psi(\iota_{\mathcal{Q}_1}x_1 - \varphi_1(x_1), \ldots, \iota_{\mathcal{Q}_n}x_n - \varphi_n(x_n))\end{array}}{-QU_k}\;(\iota_{\mathcal{Q}-}\mathsf{E}^{\mathrm{A}}_k2)$$

$$\frac{\begin{array}{c}\mathcal{D}_1\\ \psi(\iota_{\mathcal{Q}_1}x_1 - \varphi_1(x_1), \ldots, \iota_{\mathcal{Q}_n}x_n - \varphi_n(x_n))\end{array}}{-P}\;(\iota_{\mathcal{Q}-}\mathsf{E}^{\mathrm{A}}_i3)$$

where $i \in \{1, \ldots, n\}$ (arity of ψ), $k \in \{1, \ldots, n\}$

(Likewise with $-\psi$.)

Generalization A

Parallel QD: Symbolizations

(1.2) The pope blesses the bishop.
$Blesses^2(\iota_{\mathcal{P}} x(Pope(x)), \iota_{\mathcal{Q}} y(Bishop(y)))$

(1.7) The dog descends from the wolf.
$Descends\text{-}from^2(\iota_{\{Dog\}} x(Dog(x)), \iota_{\{Wolf\}} y(Wolf(y)))$

(1.8) The pope puts the zucchetto on the zucchetto.
$Puts\text{-}on^3(\iota_{\mathcal{P}} x(Pope(x)), \iota_{\mathcal{Q}'} y(Zucchetto(y)), \iota_{\mathcal{Q}''} z(Zucchetto(z)))$

Parallel QD: An analysis of (1.8)

(1.8) The pope puts the zucchetto on the zucchetto.
$P^3(\iota_{\mathcal{P}}x(P^1(x)), \iota_{\mathcal{Q}'}y(Z^1(y)), \iota_{\mathcal{Q}''}z(Z^1(z)))$

Let $\mathcal{Q}', \mathcal{Q}'' \subset \mathcal{P}$ such that $\mathcal{Q}' \neq \mathcal{Q}''$.

$$\begin{array}{ccc} \mathcal{D}_{1(E)} & \mathcal{D}_{2(E)} & \mathcal{D}_{3(E)} \\ \exists x P^1(x) & \exists y Z^1(y) & \exists z Z^1(z) \end{array} \tag{8}$$

$$\begin{array}{c} \mathcal{D}_{1(QU)} \\ \forall u_1 \forall v_1((P^1(u_1) \& P^1(v_1)) \supset u_1 \stackrel{+}{=}_{\mathcal{P}} v_1) \\ \mathcal{D}_{2(QU)} \\ \forall u_2 \forall v_2((Z^1(u_2) \& Z^1(v_2)) \supset u_2 \stackrel{+}{=}_{\mathcal{Q}'} v_2) \\ \mathcal{D}_{3(QU)} \\ \forall u_3 \forall v_3((Z^1(u_3) \& Z^1(v_3)) \supset u_3 \stackrel{+}{=}_{\mathcal{Q}''} v_3) \end{array} \tag{9}$$

Parallel QD: An analysis of (1.8) [contd.]

$$\frac{\mathcal{D}_{(P)}}{\forall w_1 \forall w_2 \forall w_3((P^1(w_1) \& Z^1(w_2) \& Z^1(w_3)) \supset P^3(w_1, w_2, w_3))} \quad (10)$$

Canonical derivation:
Let $\{\mathcal{D}_{(E)}\} = \{\mathcal{D}_{1(E)}, \mathcal{D}_{2(E)}, \mathcal{D}_{3(E)}\}$ and $\{\mathcal{D}_{(QU)}\} = \{\mathcal{D}_{1(QU)}, \mathcal{D}_{2(QU)}, \mathcal{D}_{3(QU)}\}$.

$$\frac{\{\mathcal{D}_{(E)}\} \qquad \{\mathcal{D}_{(QU)}\} \qquad \mathcal{D}_{(P)}}{P^3(\iota_{\mathcal{P}} x(P^1(x)), \iota_{\mathcal{Q}'} y(Z^1(y)), \iota_{\mathcal{Q}''} z(Z^1(z)))} \, (\iota_{\mathcal{Q}} \mathrm{I}_3^{\mathrm{A}}) \quad (11)$$

Generalization B

Parallel nested QD: Examples

(1.9) The king of the jungle loves the queen of the desert.

(1.10) John is the mayor of Pittsburgh. (Cf. [5]: (61).)

Generalization B

Parallel nested QD: Extension of $\mathcal{L}\iota$

1. *Parallel nested positive QD*:
$\psi(\iota_{\mathcal{Q}_{n_1}} x_{n_1} \varphi_{n_1}(x_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} \varphi_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1}))\ldots), \ldots, \iota_{\mathcal{Q}_{n_m}} x_{n_m} \varphi_{n_m}(x_{n_m}, \ldots, \iota_{\mathcal{Q}_{2_m}} x_{2_m} \varphi_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m}))\ldots)) =_{def}$

$(\exists x_{n_1} \varphi_{n_1}(x_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} \varphi_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1})))\&\ldots\& \exists x_{n_m} \varphi_{n_m}(x_{n_m}, \ldots, \iota_{\mathcal{Q}_{2_m}} x_{2_m} \varphi_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m}))))\&$

$(\forall u_{n_1} \forall v_{n_1}((\varphi_{n_1}(u_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} \varphi_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1})))\& \varphi_{n_1}(v_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} \varphi_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{n}(x_{1_1})))) \supset u_{n_1} \overset{+}{=}_{\mathcal{Q}_{n_1}} v_{n_1})\&\ldots\& \forall u_{n_m} \forall v_{n_m}((\varphi_{n_m}(u_{n_m}, \ldots, \iota_{\mathcal{Q}_{2_m}} x_{2_m} \varphi_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m})))\& \varphi_{n_m}(v_{n_m}, \ldots, \iota_{\mathcal{Q}_{2_m}} x_{2_m} \varphi_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m})))) \supset u_{n_m} \overset{+}{=}_{\mathcal{Q}_{n_m}} v_{n_m}))\&$

$(\forall w_{n_1} \ldots \forall w_{n_m}(\varphi_{n_1}(w_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} \varphi_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1})))\&\ldots\& \varphi_{n_m}(w_{n_m}, \ldots, \iota_{\mathcal{Q}_{2_m}} x_{2_m} \varphi_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m})))) \supset \psi(w_{n_1}, \ldots, w_{n_m})))$

2. *Parallel nested negative QD*: *mutatis mutandis*.

(Likewise with $-\psi$.)

Parallel nested QD: (E)-Abbreviations

$\{E_{1_k}\}$:

$$\underbrace{\exists x_{1_1}\varphi_{1_1}(x_{1_1})}_{E_{1_1}},\ldots,\underbrace{\exists x_{1_m}\varphi_{1_m}(x_{1_m})}_{E_{1_m}}$$

$\{E_{2_k}\}$:

$$\underbrace{\exists x_{2_1}\varphi_{2_1}(x_{2_1},\ldots,\iota_{\mathcal{Q}_{1_1}}x_{1_1}\varphi_{1_1}(x_{1_1}))}_{E_{2_1}},\ldots,\underbrace{\exists x_{2_m}\varphi_{2_m}(x_{2_m},\ldots,\iota_{\mathcal{Q}_{1_m}}x_{1_m}\varphi_1(x_{1_m}))}_{E_{2_m}}$$

$\vdots$

$\{E_{n_k}\}$:

$$\underbrace{\exists x_{n_1}\varphi_{n_1}(x_{n_1},\ldots,\iota_{\mathcal{Q}_{2_1}}x_{2_1}\varphi_{2_1}(x_{2_1},\ldots,\iota_{\mathcal{Q}_{1_1}}x_{1_1}\varphi_{1_1}(x_{1_1})))}_{E_{n_1}},\ldots,$$

$$\underbrace{\exists x_{n_m}\varphi_{n_m}(x_{n_m},\ldots,\iota_{\mathcal{Q}_{2_m}}x_{2_m}\varphi_{2_m}(x_{2_m},\ldots,\iota_{\mathcal{Q}_{1_m}}x_{1_m}\varphi_{1_m}(x_{1_m})))}_{E_{n_m}}$$

(Likewise for $\iota_{\mathcal{Q}}-$.)

Parallel nested QD: (QU)-Abbreviations

$\{QU_{1_k}\}$:

$$\underbrace{\forall u_{1_1} \forall v_{1_1} ((\varphi_{1_1}(u_{1_1}) \& \varphi_{1_1}(v_{1_1})) \supset u_{1_1} \overset{\pm}{=}_{\mathcal{Q}_{1_1}} v_{1_1})}_{QU_{1_1}}, \ldots, \underbrace{\forall u_{1_m} \forall v_{1_m} ((\varphi_{1_m}(u_{1_m}) \& \varphi_{1_m}(v_{1_m})) \supset u_{1_m} \overset{\pm}{=}_{\mathcal{Q}_{1_m}} v_{1_m})}_{QU_{1_m}}$$

$\{QU_{2_k}\}$:

$$\underbrace{\forall u_{2_1} \forall v_{2_1} ((\varphi_{2_1}(u_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1})) \& \varphi_{2_1}(v_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1}))) \supset u_{2_1} \overset{\pm}{=}_{\mathcal{Q}_{2_1}} v_{2_1})}_{QU_{2_1}}, \ldots,$$

$$\underbrace{\forall u_{2_m} \forall v_{2_m} ((\varphi_{2_m}(u_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m})) \& \varphi_{2_m}(v_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m}))) \supset u_{2_m} \overset{\pm}{=}_{\mathcal{Q}_{2_m}} v_{2_m})}_{QU_{2_m}}$$

$\vdots$

$\{QU_{n_k}\}$:

$$\underbrace{\forall u_{n_1} \forall v_{n_1}}_{QU_{n_1}} ((\varphi_{n_1}(u_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} \varphi_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1}))) \& \varphi_{n_1}(v_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} \varphi_{2_1}(x_{2_1}, \ldots,$$

$$\iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1})))) \supset u_{n_1} \overset{\pm}{=}_{\mathcal{Q}_{n_1}} v_{n_1}), \ldots, \underbrace{\forall u_{n_m} \forall v_{n_m}}_{QU_{n_m}} ((\varphi_{n_m}(u_{n_m}, \ldots, \iota_{\mathcal{Q}} x_{2_m} \varphi_{2_m}(x_{2_m}, \ldots,$$

$$\iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m}))) \& \varphi_{n_m}(v_{n_m}, \ldots, \iota_{\mathcal{Q}_{2_m}} x_{2_m} \varphi_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m})))) \supset u_{n_m} \overset{\pm}{=}_{\mathcal{Q}_{n_m}} v_{n_m})$$

(Likewise for $\iota_{\mathcal{Q}}-$.)

Generalization B

Parallel nested QD: (P)-Abbreviations

P_1:

$$\forall w_{1_1} \ldots \forall w_{1_m}((\varphi_{1_1}(w_{1_1}) \& \ldots \& \varphi_{1_m}(w_{1_m})) \supset \psi_1(w_{1_1}, \ldots, w_{1_m}))$$

P_2:

$$\forall w_{2_1} \ldots \forall w_{2_m}((\varphi_{2_1}(w_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1})) \& \ldots \& \varphi_{2_m}(w_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m}))) \supset \psi_2(w_{2_1}, \ldots, w_{2_m}))$$

$\vdots$

P_n:

$$\forall w_{n_1} \ldots \forall w_{n_m}((\varphi_{n_1}(w_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} \varphi_{2_1}(w_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1}))) \& \ldots \& \varphi_{n_m}(w_{n_m}, \ldots, \iota_{\mathcal{Q}_{2_m}} x_{2_m} \varphi_{2_m}(w_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m})))) \supset \psi_n(w_{n_1}, \ldots w_{n_m}))$$

(Likewise for $\iota_{\mathcal{Q}} - / - \psi_j$.)

Parallel nested QD: (QD)-Abbreviations

QD_1:

$$\psi_1(\iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1}), \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_{1_m}(x_{1_m}))$$

QD_2:

$$\psi_2(\iota_{\mathcal{Q}_{2_1}} x_{2_1} \varphi_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1})), \ldots,$$
$$\iota_{\mathcal{Q}_{2_m}} x_{2_m} \varphi_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_1(x_{1_m})))$$

$\vdots$

QD_n:

$$\psi_n(\iota_{\mathcal{Q}_{n_1}} x_{n_1} \varphi_{n_1}(x_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} \varphi_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} \varphi_{1_1}(x_{1_1}))\ldots), \ldots,$$
$$\iota_{\mathcal{Q}_{n_m}} x_{n_m} \varphi_n(x_{n_m}, \ldots, \iota_{\mathcal{Q}_{2_m}} x_{2_m} \varphi_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} \varphi_1(x_{1_m}))\ldots))$$

(Likewise for $\iota_{\mathcal{Q}}-/-\psi_i$.)

Generalization B

Parallel nested QD: Rules

$$\dfrac{\begin{array}{ccc}\{\mathcal{D}_{1_1}\} & \{\mathcal{D}_{2_1}\} & \mathcal{D}_{3_1} \\ \{E_{1_k}\} & \{QU_{1_k}\} & P_1\end{array}}{QD_1}\ (\iota_{\mathcal{Q}}\mathsf{I}^{\mathrm{B}}_{i,1})$$

$$\vdots$$

$$\dfrac{\begin{array}{ccc}\{\mathcal{D}_{1_n}\} & \{\mathcal{D}_{2_n}\} & \mathcal{D}_{3_n} \\ \{E_{n_k}\} & \{QU_{n_k}\} & P_n\end{array}}{QD_n}\ (\iota_{\mathcal{Q}}\mathsf{I}^{\mathrm{B}}_{i,n})$$

$$\begin{array}{c}\mathcal{D}_1 \\ \dfrac{QD_j}{E_{j_k}}\end{array}\ (\iota_{\mathcal{Q}}\mathsf{E}^{\mathrm{B}}_{k,j}1) \qquad \begin{array}{c}\mathcal{D}_1 \\ \dfrac{QD_j}{QU_{j_k}}\end{array}\ (\iota_{\mathcal{Q}}\mathsf{E}^{\mathrm{B}}_{k,j}2) \qquad \begin{array}{c}\mathcal{D}_1 \\ \dfrac{QD_j}{P_j}\end{array}\ (\iota_{\mathcal{Q}}\mathsf{E}^{\mathrm{B}}_{i,j}3)$$

where $i \in \{1, ..., m\}$ (arity of predicate in QD_j),
$j \in \{1, ..., n\}$ (level of nesting),
$k \in \{1, ..., m\}$

Mutatis mutandis, for $\iota_{\mathcal{Q}}{-}/{-}\psi_j$.

Detour conversions for parallel nested positive QD

$$\dfrac{\begin{array}{ccc}\{\mathcal{D}_{1_1}\} & \{\mathcal{D}_{2_1}\} & \mathcal{D}_{3_1} \\ \{E_{1_k}\} & \{QU_{1_k}\} & P_1\end{array}}{QD_1}\,(\iota_{\mathcal{Q}}\mathsf{I}^{\mathrm{B}}_{i,1})$$

$$\vdots$$

$$\dfrac{\begin{array}{ccc}\{\mathcal{D}_{1_n}\} & \{\mathcal{D}_{2_n}\} & \mathcal{D}_{3_n} \\ \{E_{n_k}\} & \{QU_{n_k}\} & P_n\end{array}}{\dfrac{QD_n}{E_{n_k}}\,(\iota_{\mathcal{Q}}\mathsf{E}^{\mathrm{B}}_{k,n}1)}\,(\iota_{\mathcal{Q}}\mathsf{I}^{\mathrm{B}}_{i,n})$$

conv

$$\dfrac{\begin{array}{ccc}\{\mathcal{D}_{1_1}\} & \{\mathcal{D}_{2_1}\} & \mathcal{D}_{3_1} \\ \{E_{1_k}\} & \{QU_{1_k}\} & P_1\end{array}}{QD_1}\,(\iota_{\mathcal{Q}}\mathsf{I}^{\mathrm{B}}_{i,1})$$

$$\vdots$$

$$\mathcal{D}_{1_n}$$

$$E_{n_k}$$

(Likewise for $\iota_{\mathcal{Q}-}/-\psi$.)

Detour conversions for parallel nested positive QD [contd.]

$$\cfrac{\begin{matrix}\{\mathcal{D}_{1_1}\} & \{\mathcal{D}_{2_1}\} & \mathcal{D}_{3_1} \\ \{E_{1_k}\} & \{QU_{1_k}\} & P_1\end{matrix}}{QD_1}\ (\iota_{\mathcal{Q}}\mathsf{I}^{\mathrm{B}}_{i,1})$$

$$\vdots$$

$$\cfrac{\begin{matrix}\{\mathcal{D}_{1_n}\} & \{\mathcal{D}_{2_n}\} & \mathcal{D}_{3_n} \\ \{E_{n_k}\} & \{QU_{n_k}\} & P_n\end{matrix}}{\cfrac{QD_n}{QU_{n_k}}\ (\iota_{\mathcal{Q}}\mathsf{E}^{\mathrm{B}}_{k,n}2)}\ (\iota_{\mathcal{Q}}\mathsf{I}^{\mathrm{B}}_{i,n}) \quad \text{conv} \quad \begin{matrix}\mathcal{D}_{2_n} \\ QU_{n_k}\end{matrix}$$

(Likewise for $\iota_{\mathcal{Q}-}/{-\psi}$.)

Detour conversions for parallel nested positive QD [contd.]

$$
\dfrac{\dfrac{\begin{matrix}\{\mathcal{D}_{1_1}\} & \{\mathcal{D}_{2_1}\} & \mathcal{D}_{3_1} \\ \{E_{1_k}\} & \{QU_{1_k}\} & P_1\end{matrix}}{QD_1}\;(\iota_{\mathcal{Q}}\mathsf{I}^{\mathrm{B}}_{i,1}) \\ \vdots \\ \begin{matrix}\{\mathcal{D}_{1_n}\} & \{\mathcal{D}_{2_n}\} & \mathcal{D}_{3_n} \\ \{E_{n_k}\} & \{QU_{n_k}\} & P_n\end{matrix}}{\dfrac{QD_n}{P_n}\;(\iota_{\mathcal{Q}}\mathsf{E}^{\mathrm{B}}_{i,n}3)}\;(\iota_{\mathcal{Q}}\mathsf{I}^{\mathrm{B}}_{i,n}) \qquad \text{conv} \qquad \begin{matrix}\mathcal{D}_{3_n} \\ P_n\end{matrix}
$$

(Likewise for $\iota_{\mathcal{Q}-}/-\psi$.)

Generalization B

Parallel nested QD: Symbolizations

(1.9) The king of the jungle loves the queen of the desert.
$Loves^2(\iota_{\mathcal{Q}_{2_1}} x(King\text{-}of^2(x, \iota_{\mathcal{Q}_{1_1}} y(Jungle(y)))),$
$\iota_{\mathcal{Q}_{2_2}} z(Queen\text{-}of^2(z, \iota_{\mathcal{Q}_{1_2}} u(Desert(u)))))$

(1.10) John is the mayor of Pittsburgh.
$Holds^2(John, \iota_{\mathcal{P}} x(Office\text{-}of^2(x, \iota_{\mathcal{P}} y(Mayor\text{-}of^2(y, Pittsburgh)))))$

Generalization B

Parallel nested QD: An analysis of (1.9)

(1.9) The king of the jungle loves the queen of the desert.

$$L^2(\iota_{\mathcal{Q}_{2_1}} x(K^2(x, \iota_{\mathcal{Q}_{1_1}} y(J^1(y)))), \iota_{\mathcal{Q}_{2_2}} z(Q^2(z, \iota_{\mathcal{Q}_{1_2}} u(D^1(u)))))$$

Let $\mathcal{Q}_{1_1}, \mathcal{Q}_{1_2}, \mathcal{Q}_{2_1}, \mathcal{Q}_{2_2} \subseteq \mathcal{P}$.

$\mathcal{D}_{1(E)}$	$\mathcal{D}_{2(QU)}$	$\mathcal{D}_{3(P)}$
$\exists y J^1(y)$	$\forall u_1 \forall v_1((J^1(u_1) \& J^1(v_1)) \supset u_1 \overset{\pm}{=}_{\mathcal{Q}_{1_1}} v_1)$	$\forall w_1(J^1(w_1) \supset K^2(\alpha_1, w_1))$
$\mathcal{D}'_{1(E)}$	$\mathcal{D}'_{2(QU)}$	$\mathcal{D}'_{3(P)}$
$\exists u D^1(u)$	$\forall u'_1 \forall v'_1((D^1(u'_1) \& D^1(v'_1)) \supset u'_1 \overset{\pm}{=}_{\mathcal{Q}_{1_2}} v'_1)$	$\forall w'_1(D^1(w'_1) \supset Q^2(\alpha_2, w'_1))$

Parallel nested QD: An analysis of (1.9) [contd.]

The first level of parallel nesting:

$$\mathcal{D}_{4(E)} = \cfrac{\cfrac{\mathcal{D}_{1(E)} \qquad \mathcal{D}_{2(QU)} \qquad \mathcal{D}_{3(P)}}{K^2(\alpha_1, \iota_{\mathcal{Q}_{1_1}} y(J^1(y)))} (\iota_{\mathcal{Q}} \mathsf{I}^{\mathrm{B}}_{2,1})}{\exists x(K^2(x, \iota_{\mathcal{Q}_{1_1}} y(J^1(y))))}$$

$$\mathcal{D}_{5(QU)}$$

$$\forall u_2 \forall v_2((K^2(u_2, \iota_{\mathcal{Q}_{1_1}} y(J^1(y))) \& K^2(v_2, \iota_{\mathcal{Q}_{1_1}} y(J^1(y)))) \supset u_2 \overset{+}{=}_{\mathcal{Q}_{2_1}} v_2)$$

$$\mathcal{D}'_{4(E)} = \cfrac{\cfrac{\mathcal{D}'_{1(E)} \qquad \mathcal{D}'_{2(QU)} \qquad \mathcal{D}'_{3(P)}}{Q^2(\alpha_2, \iota_{\mathcal{Q}_{1_2}} u(D^1(u)))} (\iota_{\mathcal{Q}} \mathsf{I}^{\mathrm{B}}_{2,1})}{\exists z(Q^2(z, \iota_{\mathcal{Q}_{1_2}} u(D^1(u))))}$$

$$\mathcal{D}'_{5(QU)}$$

$$\forall u'_2 \forall v'_2((Q^2(u'_2, \iota_{\mathcal{Q}_{1_2}} u(D^1(u))) \& Q^2(v'_2, \iota_{\mathcal{Q}_{1_2}} u(D^1(u)))) \supset u'_2 \overset{+}{=}_{\mathcal{Q}_{2_2}} v'_2)$$

Parallel nested QD: An analysis of (1.9) [contd.]

The second level of parallel nesting:

$$\{\mathcal{D}_{(E)}\} = \{\mathcal{D}_{4(E)}, \mathcal{D}'_{4(E)}\}$$

$$\{\mathcal{D}_{(QU)}\} = \{\mathcal{D}_{5(QU)}, \mathcal{D}'_{5(QU)}\}$$

$$\begin{array}{c}\mathcal{D}_{6(P)}\\ \forall w_2 \forall w_3((K^2(w_2, \iota_{\mathcal{Q}_{1_1}} y(J^1(y)))\&(Q^2(w_3, \iota_{\mathcal{Q}_{1_2}} u(D^1(u)))) \supset L^2(w_2, w_3))\end{array}$$

$$\frac{\{\mathcal{D}_{(E)}\} \qquad \{\mathcal{D}_{(QU)}\} \qquad \mathcal{D}_{6(P)}}{L^2(\iota_{\mathcal{Q}_{2_1}} x(K^2(x, \iota_{\mathcal{Q}_{1_1}} y(J^1(y)))), \iota_{\mathcal{Q}_{2_2}} z(Q^2(z, \iota_{\mathcal{Q}_{1_2}} u(D^1(u)))))} (\iota_{\mathcal{Q}} \mathrm{I}^{\mathrm{B}}_{2,2}) \qquad (12)$$

Parallel conjunctively nested QD: Examples

(1.11) The man wearing the beret with the button is French. ([4]: 450.)

(1.12) The man wearing the beret and carrying the newspaper is French. ([4]: 451.)

(1.13) The man wearing the beret and carrying the newspaper walks his dog.

Parallel conjunctively nested QD: CN-formulae

CN-formulae:

1. A *positive* CN-formula is either
   (a) an atomic formula,
   (b) a parallel QD-formula, or
   (c) a parallel QD-formula containing a conjunction of formulae of the form of (1a), (1b), (1c).
2. A *negative* CN-formula is either
   (a) an atomic negative predication,
   (b) a parallel negative QD-formula, or
   (c) a parallel negative QD-formula containing a conjunction of formulae of the form of (2a), (2b), (2c).

Generalization C

Parallel conjunctively nested QD: Extension of $\mathcal{L}\iota$

1. *Parallel conjunctively nested positive QD*:
$\psi(\iota_{\mathcal{Q}_{n_1}} x_{n_1} C_{n_1}(x_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} C_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} C_{1_1}(x_{1_1}))\ldots), \ldots, \iota_{\mathcal{Q}_{n_m}} x_{n_m} C_{n_m}(x_{n_m}, \ldots,$
$\iota_{\mathcal{Q}_{2_m}} x_{2_m} C_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} C_{1_m}(x_{1_m}))\ldots)) =_{def}$

$(\exists x_{n_1} C_{n_1}(x_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} C_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} C_{1_1}(x_{1_1})))\&\ldots\&$
$\exists x_{n_m} C_{n_m}(x_{n_m}, \ldots, \iota_{\mathcal{Q}_{2_m}} x_{2_m} C_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} C_{1_m}(x_{1_m}))))\&$

$(\forall u_{n_1} \forall v_{n_1}((C_{n_1}(u_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} C_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} C_{1_1}(x_{1_1})))\&$
$C_{n_1}(v_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} C_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} C_n(x_{1_1})))) \supset u_{n_1} \overset{+}{=}_{\mathcal{Q}_{n_1}} v_{n_1})\&\ldots\&$
$\forall u_{n_m} \forall v_{n_m}((C_{n_m}(u_{n_m}, \ldots, \iota_{\mathcal{Q}_{2_m}} x_{2_m} C_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} C_{1_m}(x_{1_m})))\& C_{n_m}(v_{n_m}, \ldots,$
$\iota_{\mathcal{Q}_{2_m}} x_{2_m} C_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} C_{1_m}(x_{1_m})))) \supset u_{n_m} \overset{+}{=}_{\mathcal{Q}_{n_m}} v_{n_m}))\&$

$(\forall w_{n_1} \ldots \forall w_{n_m}(C_{n_1}(w_{n_1}, \ldots, \iota_{\mathcal{Q}_{2_1}} x_{2_1} C_{2_1}(x_{2_1}, \ldots, \iota_{\mathcal{Q}_{1_1}} x_{1_1} C_{1_1}(x_{1_1})))\&\ldots\&$
$C_{n_m}(w_{n_m}, \ldots, \iota_{\mathcal{Q}_{2_m}} x_{2_m} C_{2_m}(x_{2_m}, \ldots, \iota_{\mathcal{Q}_{1_m}} x_{1_m} C_{1_m}(x_{1_m})))) \supset \psi(w_{n_1}, \ldots, w_{n_m})))$

where the C-formulae are positive CN-formulae

2. *Parallel conjunctively nested negative QD*: *mutatis mutandis*.

(Likewise with $-\psi_j$.)

Parallel conjunctively nested QD: Rules

The rules are like those for parallel nested QD, except that atomic predicates are replaced by CN-formulae.

$$\cfrac{\begin{array}{ccc}\{\mathcal{D}_{1_1}\} & \{\mathcal{D}_{2_1}\} & \mathcal{D}_{3_1} \\ \{E_{1_k}\} & \{QU_{1_k}\} & P_1\end{array}}{QD_1}\ (\iota_{\mathcal{Q}}\mathsf{I}^{\mathrm{C}}_{i,1})$$

$$\vdots$$

$$\cfrac{\begin{array}{ccc}\{\mathcal{D}_{1_n}\} & \{\mathcal{D}_{2_n}\} & \mathcal{D}_{3_n} \\ \{E_{n_k}\} & \{QU_{n_k}\} & P_n\end{array}}{QD_n}\ (\iota_{\mathcal{Q}}\mathsf{I}^{\mathrm{C}}_{i,n})$$

$$\cfrac{\begin{array}{c}\mathcal{D}_1 \\ QD_j\end{array}}{E_{j_k}}\ (\iota_{\mathcal{Q}}\mathsf{E}^{\mathrm{C}}_{k,j}1) \qquad \cfrac{\begin{array}{c}\mathcal{D}_1 \\ QD_j\end{array}}{QU_{j_k}}\ (\iota_{\mathcal{Q}}\mathsf{E}^{\mathrm{C}}_{k,j}2) \qquad \cfrac{\begin{array}{c}\mathcal{D}_1 \\ QD_j\end{array}}{P_j}\ (\iota_{\mathcal{Q}}\mathsf{E}^{\mathrm{C}}_{i,j}3)$$

where $i \in \{1, \ldots, m\}$ (arity of predicate in QD_j),
$j \in \{1, \ldots, n\}$ (level of nesting), $k \in \{1, \ldots, m\}$

Mutatis mutandis, for $\iota_{\mathcal{Q}} - / - \psi_j$.

Generalization C

Parallel conjunctively nested QD: Symbolizations

(1.11) The man wearing the beret with the button is French.
$French(\iota_{\mathcal{Q}_3}x(Man(x)\ \&\ Wears^2(x, \iota_{\mathcal{Q}_2}y(Beret(y)\ \&\ Has^2(y, \iota_{\mathcal{Q}_1}z(Button(z)))))))$

(1.12) The man wearing the beret and carrying the newspaper is French.
$French(\iota_{\mathcal{Q}_2}x(Man(x) \& Wears^2(x, \iota_{\mathcal{Q}_1}y(Beret(y)))\ \&\ Carries^2(x, \iota_{\mathcal{Q}'_1}z(Newspaper(z)))))$

(1.13) The man wearing the beret and carrying the newspaper walks his dog.
$Walks^2(\iota_{\mathcal{Q}_2}x(Man(x) \& Wears^2(x, \iota_{\mathcal{Q}_1}y(Beret(y)))\ \&\ Carries^2(x, \iota_{\mathcal{Q}'_1}z(Newspaper(z)))), \iota_{\mathcal{Q}_3}u(Dog(u) \& Owns^2(\iota_{\mathcal{Q}_2}x(Man(x) \&(Wears^2(x, \iota_{\mathcal{Q}_1}y(Beret(y))) \& Carries^2(x, \iota_{\mathcal{Q}'_1}z(Newspaper(z)))), u)))$

Parallel conjunctively nested QD: An analysis of (1.13)

(1.10) The man wearing the beret and carrying the newspaper walks his dog.

$W_1^2(\iota_{\mathcal{Q}_2} x(M^1(x) \& W_2^2(x, \iota_{\mathcal{Q}_1} y(B^1(y))) \& C^2(x, \iota_{\mathcal{Q}'_1} z(N^1(z))))),$
$\iota_{\mathcal{Q}_3} u(D^1(u) \&\ O^2(\iota_{\mathcal{Q}_2} x(M^1(x) \& (W_2^2(x, \iota_{\mathcal{Q}_1} y(B^1(y)))\ \&$
$C^2(x, \iota_{\mathcal{Q}'_1} z(N^1(z)))), u)))$

Let $\mathcal{Q}_j, \mathcal{Q}'_j \subseteq \mathcal{P}$.

$\mathcal{D}_{1(E)}$	$\mathcal{D}_{2(QU)}$	$\mathcal{D}_{3(P)}$
$\exists y B^1(y)$	$\forall u_1 \forall v_1((B^1(u_1) \& B^1(v_1)) \supset u_1 \stackrel{\pm}{=}_{\mathcal{Q}_1} v_1)$	$\forall w_1(B^1(w_1) \supset W_2^2(\alpha_1, w_1))$
$\mathcal{D}'_{1(E)}$	$\mathcal{D}'_{2(QU)}$	$\mathcal{D}'_{3(P)}$
$\exists z N^1(z)$	$\forall u'_1 \forall v'_1((N^1(u'_1) \& N^1(v'_1)) \supset u'_1 \stackrel{\pm}{=}_{\mathcal{Q}'_1} v'_1)$	$\forall w'_1(N^1(w'_1) \supset C^2(\alpha_1, w'_1))$

Parallel conjunctively nested QD: An analysis of (1.13) [contd.]

$$\dfrac{\dfrac{\mathcal{D}_4}{M^1(\alpha_1)} \quad \dfrac{\dfrac{\mathcal{D}_{1(E)} \quad \mathcal{D}_{2(QU)} \quad \mathcal{D}_{3(P)}}{W_2^2(\alpha_1, \iota_{\mathcal{Q}_1} y(B^1(y)))} (\iota_{\mathcal{Q}}\mathrm{I}^{\mathrm{C}}_{2,1}) \quad \dfrac{\mathcal{D}'_{1(E)} \quad \mathcal{D}'_{2(QU)} \quad \mathcal{D}'_{3(P)}}{C^2(\alpha_1, \iota_{\mathcal{Q}'_1} z(N^1(z)))} (\iota_{\mathcal{Q}}\mathrm{I}^{\mathrm{C}}_{2,1})}{W_2^2(\alpha_1, \iota_{\mathcal{Q}_1} y(B^1(y)))\&C^2(\alpha_1, \iota_{\mathcal{Q}'_1} z(N^1(z)))} (\&\mathrm{I})}{M^1(\alpha_1)\&W_2^2(\alpha_1, \iota_{\mathcal{Q}_1} y(B^1(y)))\&C^2(\alpha_1, \iota_{\mathcal{Q}'_1} z(N^1(z)))} (\&\mathrm{I})$$

$$\mathcal{D}_{5(E)} = \dfrac{M^1(\alpha_1)\&W_2^2(\alpha_1, \iota_{\mathcal{Q}_1} y(B^1(y)))\&C^2(\alpha_1, \iota_{\mathcal{Q}'_1} z(N^1(z)))}{\exists x(\underbrace{M^1(x)\&W_2^2(x, \iota_{\mathcal{Q}_1} y(B^1(y)))\&C^2(x, \iota_{\mathcal{Q}'_1} z(N^1(z)))}_{=A_1(x)})}$$

$$\mathcal{D}_{6(QU)}$$

$$\forall u_2 \forall v_2((A_1(u_2)\ \&\ A_1(v_2)) \supset u_2 \stackrel{+}{=}_{\mathcal{Q}_2} v_2)$$

$$\mathcal{D}_{7(P)}$$

$$\forall w_2((M^1(w_2)\&W_2^2(w_2, \iota_{\mathcal{Q}_1} y(B^1(y)))\&C^2(w_2, \iota_{\mathcal{Q}'_1} z(N^1(z)))) \supset O^2(w_2, \alpha_2))$$

Parallel conjunctively nested QD: An analysis of (1.13) [contd.]

$$\mathcal{D}_{9(E)} = \cfrac{\cfrac{\begin{matrix}\mathcal{D}_8 \\ D^1(\alpha_2)\end{matrix} \qquad \cfrac{\mathcal{D}_{5(E)} \qquad \mathcal{D}_{6(QU)} \qquad \mathcal{D}_{7(P)}}{O^2(\iota_{\mathcal{Q}_2}x(M^1(x)\&W^2_2(x,\iota_{\mathcal{Q}_1}y(B^1(y)))\&C^2(x,\iota_{\mathcal{Q}'_1}z(N^1(z)))),\alpha_2)}\,(\iota_{\mathcal{Q}}\mathrm{I}^{\mathrm{C}}_{2,2})}{D^1(\alpha_2)\&O^2(\iota_{\mathcal{Q}_2}x(M^1(x)\&W^2_2(x,\iota_{\mathcal{Q}_1}y(B^1(y)))\&C^2(x,\iota_{\mathcal{Q}'_1}z(N^1(z)))),\alpha_2)}\,(\&\mathrm{I})}{\exists u(\underbrace{D^1(u)\&O^2(\iota_{\mathcal{Q}_2}x(M^1(x)\&W^2_2(x,\iota_{\mathcal{Q}_1}y(B^1(y)))\&C^2(x,\iota_{\mathcal{Q}'_1}z(N^1(z)))),u)}_{A_2(u)})}$$

$$\begin{matrix}\mathcal{D}_{10(QU)} \\ \forall u_3 \forall v_3((A_2(u_3)\&A_2(v_3)) \supset u_3 \overset{+}{=}_{\mathcal{Q}_3} v_3)\end{matrix}$$

$$\begin{matrix}\mathcal{D}_{11(P)} \\ \forall w_3 \forall w_4((M^1(w_3)\&W^2_2(w_3,\iota_{\mathcal{Q}_1}y(B^1(y)))\&C^2(w_3,\iota_{\mathcal{Q}'_1}z(N^1(z)))) \;\& \\ (D^1(w_4)\&O^2(\iota_{\mathcal{Q}_2}x(M^1(x)\&W^2_2(x,\iota_{\mathcal{Q}_1}y(B^1(y))\&C^2(x,\iota_{\mathcal{Q}'_1}z(N^1(z)))),w_4))) \supset W^2_1(w_3,w_4))\end{matrix}$$

$$\cfrac{\mathcal{D}_{5(E)} \quad \mathcal{D}_{9(E)} \qquad \mathcal{D}_{6(QU)} \quad \mathcal{D}_{10(QU)} \qquad \mathcal{D}_{11(P)}}{\begin{matrix}W^2_1(\iota_{\mathcal{Q}_2}x(M^1(x)\&W^2_2(x,\iota_{\mathcal{Q}_1}y(B^1(y)))\;\&\;C^2(x,\iota_{\mathcal{Q}'_1}z(N^1(z)))), \\ \iota_{\mathcal{Q}_3}u(D^1(u)\&O^2(\iota_{\mathcal{Q}_2}x(M^1(x)\&(W^2_2(x,\iota_{\mathcal{Q}_1}y(B^1(y)))\;\&\;C^2(x,\iota_{\mathcal{Q}'_1}z(N^1(z)))),u)))\end{matrix}}\,(\iota_{\mathcal{Q}}\mathrm{I}^{\mathrm{C}}_{2,3})$$

Summary

- Bipredicational language:
  - predication vs. negative predication
  - qualified identity (uniqueness, definiteness)
- Bipredicational natural deduction:
  - normalization
  - subexpression (incl. subformula) property, internal completeness
- Proof-theoretic semantics for:
  - (parallel, nested) (complete, incomplete, generic) def. descriptions
  - copula+definite description, possessives
- Philosophy:
  - intuitionistic epistemology
  - nominalism wrt semantic ontology

Thank you!

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